THE SOCIAL DISCOUNT RATE AS A FUNCTION OF INDIVIDUALS' TIME PREFERENCES: ELUSIVE THEORETICAL CONSTRUCTS AND PRACTICAL ADVICE FOR BENEFIT-COST ANALYSTS

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Abstract: If benefit-cost analysts are charged with assessing whether aggregate "wealth" is increased (i.e., Kaldor-Hicks potential compensation) it requires discounting individual citizen's future benefits and costs by the citizen's own time preference discount factor. Heterogeneity in time discounting across citizens produces substantially different discounting than results from the standard practice of using a social discount rate. As shown in Gollier and Zeckhauser (2005), such heterogeneity can produce discounting that declines as the time horizon for the project lengthens (similar to Weitzman (2001), but for very different reasons). We discuss the practical and theoretical challenges that follow from incorporating individual discount factors, including the prospect for *ex post* policy regret, and provide practical advice for benefit-cost analysts given the lack of empirical evidence needed to facilitate the estimation of elusive theoretical constructs.

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The potential compensation test as recommended by Kaldor (1939) and Hicks (1939), which evaluates whether the losers from the introduction of an economic reform or public policy could be compensated by the gains of the winners such that all persons in a community are better off, forms the foundation of modern benefit cost analysis. Yet, we argue that benefit cost analysts have not taken Kaldor-Hicks seriously when the discussion has turned to dealing with benefits and costs accrued in future years.[1]

In this paper, we imagine that there is a benefit-cost analyst who has been asked to assess whether the gains made by the winners of a given policy (given by the winners' willingness to pay for the policy) are greater than the losses to the losers of the given policy (given by the loser's willingness to accept cash in exchange for the policy's adoption), and to state whether the funds used by this policy could be spent in a way that had even larger net benefits than the policy

[1] Although not central to the conclusions in this paper, we also believe that the Kaldor-Hicks potential compensation criterion has been used inappropriately and that a simple rule in which all projects with positive net present value are accepted is superiorIn this paper, when we say "Kaldor-Hicks criterion" we are envisioning a computation of whether a policy raises aggregate "wealth" (where we use wealth in a broad sense to include anything that individuals value, including moral sentiments). Further, when we use the term "Kaldor-Hicks criterion," we are assuming an ethical justification of the criterion that has the spirit of Zerbe and Scott's (2012) "Pareto Relevant Portfolio Approach." They argue that Kaldor-Hicks criterion (as traditionally used) is insufficiently justified, and further argue that Benefit Cost Analysis (BCA) should use the "Aggregate Criterion" to simply assess whether the sum of the willingness to pay for a particular policy by the winners exceeds the sum of the willingness to accept by the losers, as such a decision rule is more likely than any other rule to raise aggregate wealth and reduce the number of losers when applied across a portfolio of policy decisions.

under consideration. If these conditions are met, then the analyst would conclude that the Kaldor-Hicks criterion was met and would recommend the policy's adoption.

Traditionally, such analysts have discounted benefits received and costs incurred in future years using a social discount rate. In this paper, which builds on the insights in Gollier and Zeckhauser (2005), we evaluate what would happen if the analyst used individual-level discounting of costs and benefits (i.e., discounted the benefits received and costs incurred by individuals using the individual's own time preference discount factor), rather than using a singular social discount rate. We note the virtues and pitfalls of such an approach. We will show that there are several practical constraints that may make using individual discounting infeasible, and that using such individual discounting (even if possible) could lead to *ex post* regret making the use of individual discounting to be unwise. Finally, we note that if using individual-discounting leads the analyst to construct a social discount rate that is less than the social opportunity cost of capital, then the resulting decisions will not maximize aggregate wealth as alternative projects should be chosen. This last problem can easily be addressed by the analyst noting that such alternative projects should be undertaken rather than the proposed policy.

Traditionally, benefit-cost analysts have not made a distinction between whether the benefits and costs of the policy flow to those who are "citizens" (defined below) at the time that the analysis is conducted versus those who will be future citizens. If the analyst is to use individual discount factors in the analysis, the distinction between current and future citizens, and the decision about who has standing in the analysis, becomes important. In the next section we develop the analysis assuming that only current citizens have standing, and then we follow this section by redeveloping the analysis assuming that future citizens also have standing.

1. Standing Given Only to Current Citizens

1.1. *Derivation of Net Present Value of a Policy*

In this section, we develop the analysis of a proposed economic reform or public policy assuming that only the valuations of current citizens' matter. By "current", we mean alive today. By "citizens" we mean anyone whose valuations are counted. By "valuation", we mean individual's willingness to pay \$X in year *t* for the policy outcome in year *t* to occur (WTP), or for losers of the policy, willingness to accept \$X in year *t* to allow the policy outcome in year *t* to occur (WTA). We assume that individual citizens have valuations for various states of the world including outcomes in the future and after their death. These valuations can include altruistic sentiments towards others or towards future peoples, and can include existence values.

Suppose that individual citizens have their own discount factors for benefits they will receive (or costs they will pay) in the future, and that these discount factors vary across individuals.[2] If the analyst takes the Kaldor-Hicks criterion seriously, the analyst would compute each individual citizen's net present value of a given stream of benefits and costs, sum these net present values across citizens, and then recommend the policy if the sum of these net present values is positive. Given that the individual citizen may be dead in year t, computation of the net present value (*NPV*) of a given policy must incorporate valuations the citizen would place on the policy outcomes if the citizen is alive in year t and that value that the citizen would place on the policy outcome if the citizen were dead in year t. We denote V_{it} as citizen i's dollar valuation in year t of policy outcomes in year t under the assumption that citizen i is alive in year t. V_{it} equals citizen i's WTP in year t for the policy outcome in year t assuming that the policy has a positive net valuation to citizen i in year t, or –WTA if the policy has a negative net valuation to citizen i in year t. We denote Z_{it} as citizen i's valuation in year 0 of policy outcomes in year t under the assumption that citizen i is dead in year t. That is, Z_{it} reflects citizen i's WTP (or –WTA) today for the policy outcomes in year t (after death) to occur. The expected value of the policy outcomes in year t to citizen i today (year 0) is $\pi_{it}Z_{it}+(1-\pi_{it})e^{-r_i t}V_{it}$, where π_{it} denotes citizen i's subjective expected probability that s/he will be dead in year t, and r_i denotes citizen i's time preference rate.[3, 4] The certainty equivalent value of the risky proposition is

[2] For simplicity, we are assuming a time separable utility function. Further, we are defining the prospective discount factor as the number that when multiplied by the citizen's willingness to pay/accept for the outcome of the policy in year t would yield the equivalent dollar value in the present. Later in the paper, we define a "retrospective discount factor." When we refer to a "discount factor", we are referring to the willingness of the citizen to trade money across periods (as opposed to the willingness to transfer utility between periods).

[3] We assume here that individuals use exponential discounting. This assumption is not important for our analysis, but makes the discussion simpler. If individuals use hyperbolic discounting, or any other form of discounting the future, then $e^{-r_i t}$ can be replaced by δ_{it} which is citizen i's

$F_i(\pi_{it}, Z_{it}, e^{-r_i t}V_{it})$, where the *F*() function has greater curvature for more risk-averse individuals. The *NPV* of a given policy is given as follows, where *N* denotes the total number of current citizens[5]:

(1) $NPV_{Policy} = \sum_{i=1}^{N} \sum_{t=0}^{\infty} F_i(\pi_{it}, Z_{it}, e^{-r_i t}V_{it})$

If citizen *i* is dead in year *t*, then what is relevant for the current benefit-cost analysis is citizen *i*'s WTP/WTA today for the policy outcomes in year *t* (Z_{it}). If citizen *i* is alive in year *t*, then what is relevant for the current benefit-cost analysis is citizen *i*'s willingness to accept in year *t* to forgo the policy, discounted back to today ($e^{-r_i t}V_{it}$). This expected value of the policy outcomes in year *t* must be converted into the certainty equivalent dollar value as the adoption of the policy creates a risky outcome for the citizen. Equation 1 thus gives the aggregate net present value of the policy to current citizens.

Note how different this proposed procedure is from current practice. In standard practice, benefits and costs to those with standing are aggregated within a given year, *t*; the net present value of net benefits in year *t* is derived by multiplying by a social discount factor (typically e^{-rt}, where *r* is the chosen social discount rate); and these net present values are summed across years. The net present value of given policy in standard analyses is given as follows:

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discount factor for year *t*. We consider the implications of hyperbolic discounting later in the article. Since we are assuming that the citizen is alive with certainty when consideringV_{it}, the corresponding citizen's time preference rate should not include discounting due to the prospect of possible death but should rather simply capture the preference for consumption today versus consumption in year *t*. Uncertainty about whether the person is alive or dead in year *t* is dealt with subsequently in the *F*() function.

[4] We are assuming that $\pi_{it_{it}}$, V_{it}, Z_{it} and r_i are independent.

[5] Cameron and Gerdes (2005) consider heterogeneity similarly (although they do not consider the prospect of death in their analysis). They write: "a formula that honors individual time preferences would use individual discount rates, r_i: $PDV_i = \sum_{i=1}^{N}(\sum_{t=1}^{T} (1 + r_i)^{-t} b_{it})$ …In this case, the first step is to discount individual net benefits back to the present using a discount factor appropriate for that individual, $(1 + r_i)^{-t}$. The second step is to aggregate these individual discounted net benefits into a measure of social benefits. " (p. 4). The *i* subscript on "PDV_i" appears to be a typo as the double summation gives the social present discounted value.

(2) $NPV_{Policy} = \sum_{t=0}^{\infty} e^{-rt} \sum_{i=1}^{N} V_{it}$

The effects of policy induced risk given the probability of death on individual citizens' valuations are not considered in standard benefit-cost analysis.

1.2. *Practical Challenges in Using Equation 1*

If the benefit-cost analyst's goal is to determine whether the aggregate wealth of current citizens from their current perspective is increased by adoption of the given policy (i.e., if the Kaldor-Hicks potential compensation test is met giving no standing to future persons or non-citizens), then Equation 1 is the appropriate sum to compute. There are two broad reasons why Equation 2 is used rather than Equation 1: practical challenges in deriving Equation 1 (our first major point) and the possibility for policy regret (our second major point).

It would be a substantial practical challenge to a benefit-cost analyst to derive *NVP* in Equation 1 for any proposed project. The analyst would need to know the joint distribution of individual valuations, time preference rates, subjective probabilities of death, and risk aversion. Thus, literally calculating Equation 1 would be impossible due to the data gathering constraints. The question becomes whether an approximation of Equation 1 would be a more useful guide to policy than an approximation of Equation 2. Below we discuss how the analyst could make a practical attempt to use the ideas in Equation 1. We define an "Equity Rate" that functions like a social discount rate and uses some of the ideas in Equation 1 (incorporating heterogeneity in individual time preference rates, but dropping consideration of valuations after death and risk aversion).

It is very difficult to isolate individual's time preference rates, let alone the joint distribution of project valuations and time preference rates.[6] In a broad survey of empirically elicited discount rates, Frederick et al. (2002 Table 1) find spectacular disagreement among dozens of studies that purport to be measuring time preference—from annual discount rates of

[6] It is likewise challenging to identify individual's valuations of a project (whether they are expected to be alive or dead in year *t*). Since this challenge already exists for traditional benefit-cost analysis, we do not further discuss these empirical challenges, despite their importance.

negative 6% to infinity. The median value listed in their Table 1 is 24% with an interquartile range of 8% to 158%. They note:

> "[Table 1] reveals spectacular disagreement among dozens of studies that all purport to be measuring time preference. This lack of agreement likely reflects the fact that the various elicitation procedures used to measure time preference consistently fail to isolate time preference, and instead reflect, to varying degrees, a blend of both pure time preference and other theoretically distinct considerations, including: (a) intertemporal arbitrage, when tradable rewards are used; (b) concave utility; (c) uncertainty that the future reward or penalty will actually obtain; (d) inflation, when nominal monetary amounts are used; (e) expectations of changing utility; and (f) considerations of habit formation, anticipatory utility, and visceral influences" (p. 389).[7]

Thus isolating a mean time preference rate is difficult enough, while the "Equity Rate" (which we define below) requires estimating the *distribution* of time preference rates. Harrison et al. (2002) attempt to identify such a distribution. Based on experimental evidence, Harrison et al. (2002) find "that discount rates vary significantly with respect to several socio-demographic variables" (p. 1606). In particular, they find that discount rates are significantly lower for those with more education or who are unemployed and higher for those who are retired (controlling for categorical age indicators) or who believe they are credit constrained. These results suggest the possibility of correlation between project benefits and time preference rates for some projects that benefit particular demographic groups, which we discuss below.[8]

The mean discount rate found in Harrison et al. (2002) was 28%, well above market rates of interest. They note that "despite our extensive attempts to encourage credibility, the subjects

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[7] Subsequent studies by Chapman (2003) and Groom et al. (2005) provide compilations of recent literature on time preference and discounting, yet do not suggest a method for identifying a social discount rate.

[8] For a broader (but partial) review of findings on time preference heterogeneity, see: Alan and Browning (2010), Andersen et al. (2008), Anderson and Gugerty (2009), Barsky et al. (1997), Becker and Mulligan (1997), Benzion, Rapoport, and Yagil, (1989), Cagetti (2003), Coller and Williams (1999), Holden, Shiferaw, and Wik (1998), Krusell and Smith, Jr. (1998), Lawrance (1991), and Warner and Pleeter (2001).

might have doubted that we would actually follow through on the payments" (p. 1613). Thus, their estimate of a time preference rate may be biased upwards by incorporation of a risk premium of some unknown amount. Furthermore, variation in this risk premium by socio-demographic characteristics could have generated the observed variation in discount rates, even if there is no variation in pure time preference rates. Frederick et al.(2002) conjecture that "(i)f these confounding factors were adequately controlled, we suspect that many intertemporal choices or judgments would imply much lower—indeed, possibly even zero—rates of time preference" (p. 389). If their conjecture is correct, then the "Equity Rate" (defined below) would collapse to zero percent.

Since knowing the joint distribution of time preference rates and policy net benefits is highly unlikely (as is getting empirical estimates of valuations given the citizen is alive or dead), the analyst that wishes to approach the issue from the point of view of individual discount rates would likely make the following simplifying assumptions: (a) that individual citizens have a fixed lifetime horizon (i.e., the year of death is estimated by the individual's age, gender, etc.), (b) that Z_{it}=0 (i.e., no valuation of the project after the citizen's death), and (c) that individual valuations and individual time preference rates are not correlated among citizens.

If the analyst had knowledge of the distribution of individual time preference rates the analyst could aggregate these individual time preference rates to form a social discount rate, which we term the "Equity Rate," getting its name from the fact that each individual's discount factor is given equal weight. To derive the Equity Rate, first note that the following equality is true for N equal to infinity (and approximately true for large populations) given the assumption that individual valuations and individual time preference rates are not correlated among citizens[9]:

(3) $NPV_{Policy\_Net\_Benefits\_Year\_t} = \frac{1}{N}(\sum_{i=1}^{N} e^{-r_i t}) \times (\sum_{i=1}^{N} V_{it})$

Note that the second term in Equation 3, $(\sum_{i=1}^{N} V_{it})$, is the sum of costs and benefits to citizens occuring in year t, which is currently computed in all standard benefit-cost analyses and then discounted. We define the Equity Rate, R_t, as the social discount rate that discounts these future net societal benefits by giving equal weight to each citizen's discount factor:

[9] By making simplifying assumptions (a) and (b) listed above, Equation 1 simplifies to the following where V_{it}=0 in the years after death: $NPV_{Policy} = \sum_{i=1}^{N} \sum_{t=0}^{\infty} e^{-r_i t} V_{it}$. Simplifying assumption (c) then results in Equation 3.

(4) $e^{-R_t t} = \frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_i t}\right)$

Solving for R_t yields:

(5) $R_t = -\frac{ln\left(\frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_i t}\right)\right)}{t}$

This Equity Rate has several notable features. First, the Equity Rate asymptotes towards the time preference rate of the individual with the lowest individual preferred time preference rate as t goes to infinity, which is proven formally in the appendix. This result is not a novel finding. Gollier and Zeckhauser (2005) also conclude that "heterogeneous individual exponential discounting yields a collective discount rate that decreases with the time horizon" (879). We discuss their model further below. Weitzman (2001) also derives a social discount rate that declines as t goes to infinity. However, Weitzman's approach to determining the appropriate social discount rate is conceptually different from our approach and that of Gollier and Zeckhauser (2005). Weitzman notes that there is substantial disagreement amongst economists and benefit-cost practitioners about what discount rate to use. He argues that it would be sensible to assume that each expert has an equal chance of knowing the "correct" social discount rate and thus we should compute the present value of a public project using each individual expert's discount rate, and then compute an average of these present values. One can then back out the implied social discount rate. He notes:

> "What is the expected value today of an extra expected dollar at time *t*? It should be the expected present discounted value of a dollar at time *t*, weighted by the 'probability of correctness' or the 'probability of actuality' of the rate at which it is being discounted" (p. 264).

Weitzman infers that the probability that an individual expert is "correct" is given by the distribution of responses to a survey he conducted of 2,160 Ph.D.-level economists. From the survey responses, Weitzman finds that the distribution of the preferred discount rates roughly corresponds to a gamma distribution with a mean of 3.96% and standard deviation of 2.94%. Assuming a gamma distribution, Weitzman derives an implied effective discount rate of $\mu/(1+t\sigma^2/\mu)$, where μ is the mean, σ^2 is the variance, and t is the number of years in the future when the benefit is to be received (or costs paid). There are a couple of key points to note from this formula. First, if there is no variance in preferred discount rates ($\sigma^2=0$), then the appropriate social discount rate would be given by the mean preferred discount rate (μ) and this rate would be constant for all t. Second, if there is variance in the preferred discount rates, then the implied effective discount rate falls to zero as t goes to infinity.[10]

Based on the mean and standard deviation from his survey results, and assuming a gamma distribution, Weitzman concludes that the government should use a discount rate of "about" 4% for benefits/costs incurred 1-5 years hence, "about" 3% for benefits/costs incurred 6-25 years hence, "about" 2% for benefits/costs incurred 26-75 years hence, "about" 1% for benefits/costs incurred 76-300 years hence, and "about" 0% for benefits/costs incurred 301+ years hence (p. 261).[11] This result is startling: one should essentially not discount benefits if they are received more than 300 years from now! The intuition for this result is the following: if we are uncertain about which expert has the "correct" social discount rate, by averaging the near zero present values of far-future projects from experts who prefer social discount rates greater than zero with the much, much larger present values from experts who prefer a social discount rate of 0% we arrive at a weighted average implied social discount rate of nearly 0%.

The Equity Rate has the same nature as the Weitzman rate (i.e., asymptoting to the minimum individual rate). However, our rates are motivated quite differently, with Weitzman's rate being motivated by uncertainty across experts, and our rate being motivated by extending the principle of consumer sovereignty and the Kaldor-Hicks criterion to incorporate heterogeneity across citizen's preferred time preference rates.

There are some other notable implications of the Equity Rate. First, note that at longer time horizons, the total value of a government program is dominated by the valuation of a

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[10] This implied effective discount rate asymptotes towards the minimum value supported by the gamma distribution, which is zero. It is worth noting that three of the respondents to his survey gave negative time discount rates (with the minimum value being -3%). By assuming a gamma distribution, Weitzman's implied effective discount rate asymptotes towards zero rather than -3%. Further note that it is irrational for any individual to truly use a negative discount rate as this would imply the person would be willing to pay \$X to receive less than \$X in the future, when if the person set the same \$X aside they would have \$X in the future.

[11] Note that this declining social discount rate is not the result of individual preferences for hyperbolic discounting. Rather, Weitzman's survey forces the respondent to produce a single discount rate for all t that would be used by a government which uses exponential discounting.

minority of individuals with low time preference rates. Consequently, programs that have benefits in the future that would be approved by a benefit-cost analyst will be overwhelmingly rejected by the public in a referendum. To illustrate this point, suppose there exists a 10-person society with uniformly distributed time preference rates between 1% and 10%. The government proposes a policy that will pay benefits *t* years in the future and each person in society is willing to pay $1 in year *t* for those benefits. Finally, suppose that the cost of the policy must be paid today and just happens to exactly equal the sum of the 10 individual's present values. In this scenario, the costs paid today exactly equal the present value of the benefits. Table 1 shows the results.

[Insert Table 1 here]

In the third column, we show the results for a policy that would pay benefits one year from today that each citizen would value at $1 in year 1. Given the individual's different time preference rates, the present value of this $1 ranges from $0.90 to $0.99. The sum of the 10 individual's present values is $9.47. If the policy costs exactly $9.46, then a benefit-cost analyst would say that the policy passes the Kaldor-Hicks test and should thus be approved (and the analyst would be indifferent if the policy cost $9.47 today). However, if we paid for the policy with a lump sum tax of $0.947, only the first five persons would be willing to vote in favor of the policy if it were included on a referendum (as shown in the bolded entries in Table 1) because only these first five persons have present valuations that exceed this lump sum tax. As the time horizon for benefits lengthens, the share of the population willing to vote in favor of projects that pass the Kaldor-Hicks test dwindles, and we get a divergence between projects that would be approved by a benefit-cost analyst and projects that would be approved in a referendum.[12] At *t*=500, only the first person has a present valuation of the future benefit that exceeds the lump sum tax that each citizen would be required to pay. This result stems from the fact that nearly all of the total willingness to pay is coming from person 1; as shown in the second to last row of

[12] We can extend this example by assuming that discount rates in the public follow a gamma distribution with Weitzman's (2001) parameters. We simulate such a distribution for one million hypothetical individuals, and then compute the share of the public willing to vote in favor of a public project that passes the Kaldor-Hicks test (i.e., those whose valuations are greater than the average present value). This share falls rapidly: 59% (benefits 1 year hence), 55% (10 years), 41% (50 years), 30% (100 years), 9% (500 years), and 4% (1,000 years).

Table 1, the share of total willingness to pay from the project coming from person 1's present value of $1 rises from 10.5% (*t*=1) to 99.995% (*t*=1,000).

These results illustrate another major point: that heterogeneity in individual time preference rates, regardless of the distribution, is going to lead to a divergence between the median voter's preferences and the decision made by a benefit-cost analyst who evaluates gains in aggregate wealth using individual discount factors, particularly for projects with benefits further in the future. Finally, note that the Equity Rate (shown in the bottom row of Table 1) converges toward person 1's time preference rate of 1% as *t* goes to infinity. The implication is that the social discount rate that is used in benefit cost analyses should decline as the benefits and costs horizons lengthens.

1.3. *Policy Regret from Using Equation 1: One Might Incorrectly Reject Policies Without Actual Transfers*

Our second major point is that even if we were able to know the joint distribution of discount factors and project benefits and costs across citizens, there are several ways in which using Equation 1 could lead to making policy decisions that we would later regret – even if the analyst had full information on the joint distribution of valuations, discount factors, subjective probabilities of death, and risk aversion. First, given any degree of correlation between citizens' discount factors and net benefits (or even simply a finite population) and given that some citizens receive negative net benefits in future years, policies might require *actual transfers* in the future to satisfy the Kaldor-Hicks criterion in the present. Absent such transfers, we would regret the policy choices we made today. To illustrate this fact, we suppose there exists a two-person society consisting of Bill and Martha, who each use exponential discounting, but with different rates: Bill discounts the future at 7% per year, while Martha discounts future benefits at only 1% per year. Suppose that a public project will produce net benefits of $5 to Bill ten years hence and $2 to Martha – these are the future values, denoted *FV* in the equations below. Further, suppose Bill and Martha each subjectively believe that there is zero chance they will be dead ten years hence. What is the most that Bill and Martha should be willing to pay today for this project, and what implicit Equity Rate emerges from this choice? Equations 6 and 7 answer these questions:

$$(6)\quad PV_{Bill} + PV_{Martha} = FV_{Bill}\exp(-r_{Bill}t) + FV_{Martha}\exp(-r_{Martha}t)$$
$$= 5 \times e^{-7\% \times 10} + 2 \times e^{-1\% \times 10} = \$2.48 + \$1.81 = \$4.29$$

$$(7)\ (FV_{Bill}+FV_{Martha})\mathrm{e}^{-R_t t} = PV_{Bill}+PV_{Martha}$$

$$(FV_{Bill}+FV_{Martha})\mathrm{e}^{-R_t t} = FV_{Bill}\mathrm{e}^{-r_{Bill}t} + FV_{Martha}\mathrm{e}^{-r_{Martha}t}$$

$$\mathrm{e}^{-R_t t} = \frac{FV_{Bill}}{(FV_{Bill}+FV_{Martha})}\mathrm{e}^{-r_{Bill}t} + \frac{FV_{Martha}}{(FV_{Bill}+FV_{Martha})}\mathrm{e}^{-r_{Martha}t}$$

$$R_t = \frac{-\ln\left(\frac{FV_{Bill}}{(FV_{Bill}+FV_{Martha})}\mathrm{e}^{-r_{Bill}t} + \frac{FV_{Martha}}{(FV_{Bill}+FV_{Martha})}\mathrm{e}^{-r_{Martha}t}\right)}{t}$$

Equation 7 backs out the Equity Rate that would be used by a society that wanted to take the beneficiary's time preference rates into account. As the last line of Equation 7 shows, the Equity Rate is a function of the weighted average of beneficiary's discount factors, with the weights reflecting the citizen's share of net benefits. For a multi-person society, Equation 7 can be generalized as follows:

$$(8)\ R_t = \frac{-\ln\left(\sum_{i=1}^{N}\left(\frac{FV_{it}}{\sum_{i=1}^{N}FV_{it}}\right)\mathrm{e}^{-r_i t}\right)}{t}$$

Gollier and Zeckhauser's (2005) reach a similar conclusion, but based on a different (and modestly more limiting) set of assumptions. Their model is designed to help a representative agent evaluate a policy that would have current costs and future benefits (higher aggregate income) that are homogenous across citizens, but with heterogeneity across citizens in rates of impatience (i.e., discounting of future utility) and in degrees of risk aversion. They "show that the rate of impatience of the representative agent equals a weighted mean of individual rates of impatience. These weights are proportional to the individual tolerances for consumption fluctuations" (880). In their model, individuals who have less risk aversion are willing to accept more variance in their consumption over time, and thus get more benefit from a policy that raises future income at the expense of current consumption. These less risk averse individuals thus get more weight in the discount rate derived by the representative agent. Our model, which allows for heterogeneous costs and benefits for any reason, has the more general conclusion that individuals that get a greater share of the benefits (or costs) from the policy in year *t* will receive a greater weight on their discount factors when deriving the social discount rate for that year.

As we noted earlier, computation of this Equity Rate by a benefit-cost analyst would be challenging as it would require knowledge of the joint distribution of project net benefits and time preference rates. If the individual project net benefits and time preference rates were uncorrelated, then as *N* goes to infinity (i.e., for large societies) the Equity Rates given in Equations 8 and Equation 5 become equivalent.

If there is heterogeneity across citizens in benefits/costs received in year *t*, and if net benefits are positive for some citizens and negative for others in year *t*, then the Equity Rate for year *t* can become undefined. For example, suppose a project yielded \$3 of net benefit to Bill ten years hence, but \$2 of net cost to Martha ten years hence, and the analyst would like to compute R_{10}. The present value of Bill's net benefit is \$1.49, while the present value of Martha's net benefit is -\$1.82 using each person's time preference rate. Going back to the first line of Equation 7, there is no Equity Rate which can rationalize a positive sum of future values and a negative sum of present values. This conflict raises an interesting question: should such a project be approved as having positive net societal benefits? On the one hand, ten years hence the project produces positive net benefits and therefore passes the Kaldor-Hicks potential compensation test. On the other hand, the present value of the project is negative. Thus, there is no way for Bill to compensate Martha in the present when the policy choice is decided so that they are both better off in the present, without a contingent contract between Bill and Martha. In this case, if the project does not include actual compensation from Bill to Martha ten years hence (either with the project requiring taxes be collected from Bill that would be transferred to Martha in year ten, or requiring Bill to sign a contingent contract that would require Bill to compensate Martha directly in year ten), then the benefit-cost analyst would reject the project, despite the fact that Bill and Martha will regret that decision ten years hence. This seemingly irrational decision made by the benefit-cost analyst is an interesting anomaly. That is, if a benefit-cost analyst were to jointly estimate the distribution of time preference rates and net benefits, the resulting decisions could be inappropriate. This anomaly disappears for large populations when the spread of time preference rates is random with respect to future benefits/costs. The likelihood of this anomaly is higher when the aggregated future value is close to zero and when there is more positive correlation between individual benefits/costs and time preference rates. Clearly this anomaly cannot occur where there is a single social discount rate used (i.e., using standard practice). For example, if a 7% social discount rate were used the present values of Bill and Martha's future values would be 1.49 and -1.02 respectively, and the Kaldor-Hicks criterion is satisfied in the present. This anomaly depends on using the potential compensation test as the BCA test.

For reasons we discuss above, we think it would be next to impossible to obtain an estimate of the joint distribution, and thus this potential problem is most likely moot as a practical matter. Obtaining an estimate of the Equity Rate described in Equation 5 is more feasible, and even if flawed would be better than current practice of ignoring heterogeneity in time preference rates.

1.4: *Policy Regret from Using Equation 1: Should We Really be Focusing on the Present Value of the Policy and the Problem of Elusive Theoretical Constructs Resulting from the Gollier-Weitzman Puzzle*

Gollier (2004) shows that if investment projects were ranked according to their expected net future value (rather than their net present value) and if there is uncertainty regarding the future risk-free rate of return of capital, then a central planner should use a social discount rate that is ***increasing*** in t. The divergence between this result and the results in Weitzman (1998, 2001) has come to be known as the Gollier-Weitzman Puzzle. We re-illustrate this puzzle for the case of heterogeneity across citizens in their discount factors. We illustrate a correct theoretical way to handle this puzzle resulting from heterogeneous discount factors, yet note that this solution rests on theoretical constructs that are nearly impossible to measure.

To illustrate, suppose a project is considered that would impose a nominal cost on both Bill and Martha of \$80 in Year 0 (now) and each would receive a nominal benefit of \$100 in Year 1 (one year hence).[13] What is the social value of this project? If we consider the project from the perspective of now, we would discount the future benefits received by Bill and Martha using their respective discount factors and compare these to the present costs. Assume Bill and Martha each discount nominal values received in later year exponentially with the following discount rates r_{Bill} = 30% and r_{Martha} = 1%. From Bill's perspective in Year 0, this project results in net loss of \$6 (-\$80 + $e^{-0.30}$×\$100 = -\$6). From Martha's perspective in Year 0, this project results in net gain of \$19 (-\$80 + $e^{-0.01}$×\$100 = \$19). The social value of this project from the perspective of Year 0 is \$13 (i.e., -\$6 + \$19 = \$13).

While we have considered the project from the perspective of the present, we could just as easily compute the social value of the project from the viewpoint of Bill and Martha one year

[13] When we say nominal benefit (cost), we mean the citizen's willingness to pay (accept) at that moment for the outcome of the project at that moment.

hence. One year hence, Bill and Martha will value the benefits based on their nominal valuations of the benefits. The tricky thing is to consider how they value the costs paid in Year 0 one year hence. We start by assuming that Bill and Martha *always* prefer dollars received earlier to dollars received later and that the discount rates discussed above hold as Bill and Martha look backwards (i.e., as they consider the project retrospectively). This assumption means that the Year 1 valuation of the Year 0 nominal costs equals \$108 for Bill ($e^{0.30}$×\$80 = \$108) and \$81 for Martha ($e^{0.01}$×\$80 = \$81). From the Year 1 perspective, the social value of the project is \$11 ((-\$108 + \$100) + (-\$81 + \$100) = \$11).

Changing the discount rates or the nominal costs and benefits, one can easily produce an example where the project is a net winner from the perspective of Year 0 and net loser from the perspective of Year 1 (or vice-versa). The fact that the social present value of the project and the social future value of the project can produce different judgments of the merits of the project is similar to the Gollier-Weitzman Puzzle. However, the Gollier-Weitzman Puzzle emerges from a consideration of heterogeneity across experts in their estimation of the “correct” social discount rate. It is the uncertainty in the correct social discount rate that generates their heterogeneity. Gollier and Weitzman (2011) solve the Gollier-Weitzman Puzzle for this case of heterogeneity across experts by including a risk adjustment that modifies the weights used in computing the weighted-average discount factor. Unfortunately, their solution, which addresses uncertainty in the future marginal product of capital, cannot fix the puzzle in our model which has no uncertainty but rather has heterogeneity generated by differences in preferences. Thus, we are left with the unsatisfying paradox.

We believe that it would be reasonable for a central planner (or benefit-cost analyst) to respond to the puzzle in our example by *equally valuing* the perspectives of citizens who are looking prospectively at a project before its initiation and citizens who are looking retrospectively at a project’s conclusion. In the simple example above, the planner/analyst would conclude that the social value of the project is the average of the social value of the project from the perspective of Year 0 and the social value of the project from the perspective of Year 1, that is (\$13 + \$11)/2 = \$12.

Extending this line of thinking produces results which at first will seem strange, but we believe are appropriate given the planner’s choice to equally value the future perspectives of current citizens. There is no reason to limit the central planner to only consider the social value of the project to Bill and Martha from the perspective of Year 0 and Year 1. Suppose Bill and Martha will be alive in Year 2. In Year 2, they will have retrospective valuations of the project. Bill will look back at the project as having cost him $8 as of Year 1. If he persists in preferring money received earlier to money received later, and continues to exponentially discount future valuations using r_{Bill} = 30%, then he would be indifferent between an $8 loss in Year 1 and an $11 loss in Year 2. Thus, when Bill looks back at the project in Year 2, he will consider the project to have lost him $11. Martha would be indifferent between her $19.20 gain in Year 1 and a $19.39 gain in Year 2. Thus, from Martha's perspective in Year 2, the project would have a value of $19.39. The project would have a social value of $9 from the perspective of Year 2, and the social planner should equally consider the valuations of Bill and Martha from their perspectives in Year 0, Year 1, and Year 2, resulting in a social value of $11. Table 2 extends this logic assuming that Bill and Martha live for 10 years after the initiation of the project. We show that the project shifts from having positive to negative net social value four years after its initiation. This project would have larger and larger net losses as the planner extended the number of years for which a social valuation would be computed and included in the averaging, which occurs because Bill's net loss gets more magnified in time than Martha's net gain given their respective discount rates. The bottom rows of Table 2 show that the project has positive net social value if Bill and Martha each live 2 years (and have no bequest motive) and has negative net social value if Bill and Martha each live 10 years.

[Insert Table 2 here]

One thing that may seem strange in this analysis is the assumption that Bill would be indifferent between a $119 loss in Year 10 and an $8 loss in Year 1 when Bill is thinking about it in Year 10. This analysis assumes that Bill's *retrospective discount rate* is 30% (i.e., that Bill would be willing <u>in year *t*</u> to trade $X in year *t* for $e^{-0.30\times(t-y)}$×$X in year *t*-*y*). The problem is that, in the absence of a time machine, there does not exist any markets that allow an individual to transfer resources from themselves now to their former self. Economists are by training prone to assume that individuals are prospectively and retrospectively rational and should be willing to trade resources between themselves and future and former selves based on market opportunities (i.e., risk-free borrowing and lending interest rates). Thus, if Bill is prospectively rational, and Bill's prospective discount rate of 30% exists because Bill has the (spectacular!) opportunity to invest with a risk-free 30% rate of return, then in year *t*-*y*, he should be indifferent between e^{-}

$^{0.30\times(t-y)}\times$\$X received in year *t-y* and \$X received in year *t*. Likewise, when Bill arrives at year *t*, if Bill is retrospectively rational and assumes that his former self would invest funds in year *t-y* at this 30% rate, he should be indifferent between \$X received in year *t* and $e^{-0.30\times(t-y)}\times$\$X received in year *t-y*.

The lack of a market to verify the validity of this assumption regarding retrospective discount rates presents a very large problem for constructing the social value of the project in future years. We are unaware of any literature that has tried to estimate retrospective discounting and we would guess that such a literature does not exist because (a) the daunting challenge of performing such an estimation using contingent valuation surveys or constructing a believable experiment and (b) the lack of a good reason to do so. Our paper may provide future researchers a motivation to try and create such an estimate.

In contrast to this assumption, we suspect that humans are likely to favor money received now to money received later *and earlier*. That is, we suspect that individuals discount the future and the past. Philosophers throughout the ages, including Plato and Hume, have suggested that we may have limited connection to our future and past selves. Parfit (1971, 1984) further provides a philosophical argument that such limited connections can provide a *reason* to discount:

> "My concern for my future may correspond to the degree of connectedness between me now and myself in the future ... since connectedness is nearly always weaker over long periods, I can rationally care less about my further future." (Parfit, 1984, p. 313).[14]

Lack of connection with one's prior self could provide a reason to prefer money received now to money received in the past. If individuals discount money received in the past, that would fundamentally change the analysis shown above in terms of the valuations. We now assume Bill and Martha each exponentially discount future ***and prior*** nominal values. Table 3 shows the results.

[Insert Table 3 here]

[14] As cited in Frederick (2003). Frederick (2003) provides additional discussion of the history of philosophical thought on this issue, empirical evidence that individuals do feel some degree of lack of connection with their prior and future selves, but no evidence that individuals' degrees of perceived connectedness with future selves are correlated with their *prospective* discount rates.

Note that the Year 0 valuation of the project is unchanged because Year 1 benefits are still discounted the same. The Year 1 valuation of the project is now positive and large. The reason for this result is that in Year 1, both Bill and Martha are discounting (rather than inflating) the costs paid in Year 0. It's like they are treating the costs they paid as "water under the bridge." Those costs hurt them, but the pain is receding, while the benefits are strongly palpable. In Year 2, the retrospective valuation of the project is positive, but less so than from the perspective of Year 1. Both Bill and Martha look back on the project with fondness, but their memory is less acute and their positive valuation of the project is lessened. An analyst who valued each of these three valuations equally, would conclude that the social value of the project is ($13 + $62 + $51)/3 = $42.

To generalize the above examples, using the same simplifying assumptions used to derive Equation 3, we now present the net social value (NSV_t) of the project's cost and benefits that are accrued in year t by citizen i (V_{it}):

$$(9)\ NSV_t = \frac{1}{Y}\sum_{y=0}^{Y}\left(\frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_{ity}\times(t-y)}\right)\times\left(\sum_{i=1}^{N} V_{it}\right)\right)$$

In Equation 9, t denotes the year of the project under consideration and is measured by the number of years from the beginning of the project and y denotes the year from which citizen's perspective is being drawn and is measured by the number of years from the beginning of the project. Thus, for example, if t=1 and y=2, that would indicate that we are considering the value of the nominal net benefits of the project accrued in year 1 from the retrospective perspective of citizens in year 2. In Equation 9, we are computing the value of the project's net benefits in year t from the perspective of all years in which current citizens are expected to be alive, and averaging these values across years y=0 to $y=Y$, where Y is the last year in which any current citizen will be alive. r_{ity} denotes citizen i's "discount" rate for nominal net benefits received in year t considered from the perspective of year y. When $y < t$, we assume r_{ity} is positive for all citizens, which reflects the familiar method of using a discount rate for citizen i to discount nominal values received in future years. When $y = t$, we set $r_{ity} = 0$ (i.e., there is no discounting of valuations when the year under consideration is the same as the year from which citizen's perspective is being drawn). When $y > t$, if the citizen prefers dollars received in the present to dollars received in the future *and past*, then $r_{ity} > 0$ (that is, the citizen discounts nominal values received in years after *and before* year y). Instead, if $y > t$ and the citizen always prefers dollars received in earlier years to dollars received in later years, then $r_{ity} < 0$ (which will lead to inflating of nominal values received in year t). Consistent with the simplification assumed in deriving Equation 3 of no valuation of the project after death (i.e., no bequest motive), let r_{ity} be $+\infty$ if the person is expected to be dead in year y.

The Equity Rate is defined as follows:

$$(10) \qquad e^{-R_t t} = \frac{1}{Y}\sum_{y=0}^{Y}\left(\frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_{ity}\times(t-y)}\right)\right)$$

Solving for R_t yields:

$$(11) \qquad R_t = -\frac{ln\left(\frac{1}{Y}\sum_{y=0}^{Y}\left(\frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_{ity}\times(t-y)}\right)\right)\right)}{t}$$

While we argue that this Equity Rate is the correct theoretical construct given the planner's choice to equally value the future perspectives of current citizens, it represents an enormous challenge to empirically estimate. One would need to know the distribution of r_i prospectively *and retrospectively* (as well as the joint distribution of these discount rates with the years of remaining life for each person).

As before, it can be easily shown that R_t asymptotes to the minimum value of r_{ity} (for $y < t$). However, R_t can be increasing or decreasing in time (i.e., $\frac{\partial R_t}{\partial t} \lesseqgtr 0$) depending on the value of Y and whether r_{ity} is positive or negative when y > t. More distressingly, the Equity Rate can be *negative*. This seemingly nonsensical result occurs when individuals always prefer money earlier to later (even retrospectively), and Y is high and t is low. For example, consider a project that produces a \$100 nominal benefit for Bill one year hence and assume the planner sets $Y = 10$. Bill would value this project at \$74 from the perspective of now, \$100 from the perspective of one year hence, \$135 from the perspective of two years hence, …, and \$1,488 from the perspective of ten years hence. If the planner weighs each of these perspectives equally, the present social benefit of Bill receiving \$100 one year hence is \$503! Clearly, something seems amiss.

Alternatively, we believe that it would be equally reasonable for a planner or analyst to use individual discount factors to discount each citizen's future perspectives. That is, it would be reasonable for the planner to treat each citizen's future selves the way that citizen would like their future selves to be treated (a variant of the Golden Rule). If citizens always prefer money received earlier to money received later with a consistent retrospective and prospective discount factor (as in Table 2), the planner's problem becomes very simple. In this case, it is easy to show that individually discounted future perspectives all return the same values as shown in the Year 0 row of Table 2 (e.g., Bill's present value of future Bill's valuation of the project is -$6 for all future years). Thus, the planner could ignore future perspectives in computing the social value of the project.

However, if citizens always prefer money received now to money received earlier or later (as in Table 3), then future perspectives cannot be ignored. In Table 4, we show how the results in Table 3 are modified by individual discounting of future perspectives. For example, from Table 3, we reported that Bill's value of the project from the perspective of Year 2 is $30. In Table 4, we compute Bill's present value of that $30 as $17 (i.e., $e^{-0.30\times 2}\times\$30 = \$17$). What becomes clear from this analysis is that in the present, Bill would put little value on his valuations of the project from the perspective of far future years. In contrast, given her low discount rate, Martha's present value of the project from the perspective of future years remains high.

[Insert Table 4 here]

Given (a) the lack of an obvious choice between valuing citizens' future perspectives as equal to their present perspective or discounting their future perspectives by their own discount factors, (b) the theoretical ambiguity of whether the social discount rate should be increasing or decreasing in time as a result of heterogeneity in individual discount factors, and (c) the lack of empirical evidence on the nature and distribution of retrospective discounting makes it difficult to give advice to the benefit-cost practitioner on how and whether to consider individual discounting heterogeneity. In the conclusion we provide such practical advice.

1.5 *Policy Regret from Using Equation 1: Irrational Individual Discount Factors Create Irrational Policy Choices*

We now ignore the Gollier-Weitzman Puzzle and return to consider the analyst as charged with computing just the present value of the project using citizens' current perspectives. The existing literature finds that individual prospective time preference rates are not equal to market rates of interest, despite this result seeming to be incongruous with rational decision making.[15] Nonetheless, the existing literature from both field studies and lab experiments

[15] For example, suppose Bill could borrow and lend, *with certainty*, at a 2% interest in the market. It would seem foolish of Bill to forgo an offer to receive a certain $100 one year hence

imply that individuals do forgo such arbitrage possibilities. Frederick et al.(2002) conclude that "(b)ecause imputed discount rates do not, in fact, converge on the prevailing market interest rates, but instead are much higher, it seems that many respondents are neglecting capital markets and basing their choices on some other consideration" (p. 381). The failure of individuals to engage in these arbitrage possibilities calls into question the wisdom of basing collective decision making on individual behavior that could be described as irrational and inefficient. We, however, do not see these seemingly irrational individual time preference rates to be problematic from the Kaldor-Hicks perspective. Traditional benefit-cost analysis accepts individual's valuations of goods without questioning the rationality of those valuations – we should likewise accept individual's present values of future goods without questioning these citizen's methods for deriving their individual present value. Thus, if we take Kaldor-Hicks seriously, then the benefit cost analyst should not be concerned by individual discount rates differing from the market interest rate of risk-free assets.[16]

On the other hand, individual hyperbolic discounting does present a problem. If individuals practice hyperbolic discounting, the traditional assumption in the literature is that this hyperbolic discounting leads to *ex post* regret – individuals wish they would have been more patient in the past (Kirby and Herrnstein, 1995).[17] If we were to base public decision making on an aggregate of individual hyperbolic discount factors (using the Equity Rate derived from these discount factors), the resulting public decision would lead to collective regret. That is, the

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for a payment of \$90 today (as Bill could borrow the \$90 today from the market with a required repayment of \$90×1.02 = \$91.80, less than the \$100 Bill will receive with certainty).

[16] Citizens who have time preference rates below the risk-free economic opportunity cost of private investment would, however, regret the government investing in projects that have returns below this risk-free interest rate, as the government could have forgone the project and invested the funds in the market returning these citizens more in the future. Thus, *ex post* regret would occur for these overly patient citizens. We discuss this issue further in Section 1.6.

[17] Farmer and Geanakoplos (2009) present a model where hyperbolic discounting is rational if "agents cannot be sure of their own future one-period discount rates" (p. 1). If this uncertainty is the source of individual's hyperbolic discounting, then the concerns we discuss here are no longer valid.

benefit-cost analyst would reject investment in public projects that the contemporaneous citizens will later regret.

1.6. *Policy Regret from Using Equation 1: Individual Discount Factors Don't Include the Cost of Capital*

Burgess and Zerbe (2011) discuss a variety of traditional approaches to derive the social discount rate, and favor the "social opportunity cost of capital (SOC) approach, which proposes that the discount rate reflects the social (economic) pre-tax and after tax rates of return, and, in an open economy, the marginal cost of foreign funding, where the weights reflect the proportions of funding that are obtained from displaced investment, postponed consumption, and incremental funding from abroad when the government borrows to finance the project" (pp. 1-2). They conclude that the real SOC lies in the range of 6-8%.[18, 19] The main insight of the SOC is that when funds are extracted from the economy to pay for a government project, there is an opportunity cost to the society equal to the value of the alternative use of the funds (for consumption and investment). That is, the SOC rule is that government should not invest in a project that pays less than alternatives.

The Equity Rate derived from individual time preference rates will fall below the SOC if there exists citizens whose time preference rates are below the SOC (e.g., below 6-8%). If the benefit-cost analyst based conclusions solely on the Equity Rate without consideration of the SOC, then the projects that are funded that have long-term effects may be regretted by current citizens, as these citizens would have benefitted more by not extracting the funds from the

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[18] While it is true that for projects that mainly displace consumption the SOC would be lower, it is difficult and costly to separate out the consumption and capital burden for individual projects and this practice would lead to different rates for each project, which may be undesirable given the cost.

[19] The portions of the SOC that are derived from the consumption rate of interest and the marginal cost of incremental foreign funding likely incorporate a risk premium to account for the possible of death of the investor (Bruce, 2001). Given that we are seeking to discount V_{it} under the assumption of fixed lifetime horizon, we would want the SOC to be computed excluding this risk premium. As a result, the relevant measure of the appropriate SOC given our assumptions would be slightly below Burgess and Zerbe's estimated range.

economy for the government project and rather allowed the funds to be used for consumption and investment. That is, such current citizens would regret decisions made solely based on the Equity Rate.

This problem can be fixed simply by either (A) setting the Equity Rate to be equal to $max\left\{SOC, -\frac{ln\left(\frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_i t}\right)\right)}{t}\right\}$, or (B) having the analyst simply note that when project that is recommended using the Equity Rate fails using the SOC. A third, more difficult approach, would be to include displaced consumption and capital as policy outcomes, value these outcomes directly, and then discount using the Equity Rate. We recommend approach (A), with one modification. Given that there may be uncertainty across experts in the correct value of the social opportunity cost of capital, and given the persuasive arguments made by Weitzman regarding the notion of averaging the discount factors of experts, we recommend the following social discount rate: $max\left\{-\frac{ln\left(\frac{1}{J}\left(\sum_{j=1}^{J} e^{-SOC_j t}\right)\right)}{t}, -\frac{ln\left(\frac{1}{N}\left(\sum_{i=1}^{N} e^{-r_i t}\right)\right)}{t}\right\}$, where SOC_j is the estimate of the social opportunity cost of capital by expert *j*, and assuming that there are *J* experts.[20]

To illustrate this recommendation graphically, we make the following assumptions. First, we assume that the distribution of experts' estimates of the social opportunity cost would lie uniformly between 5% and 10%.[21] Second, we assume that the distribution of citizen's time preference rates lie uniformly between 0% and 30%.[22] Figure 1 demonstrates the results.

[20] Note that the first term in brackets differs from Weitzman's approach in that we would recommend asking experts for their judgment on the SOC rather than on the appropriate social discount rate. Further, we use the term "experts" to mean those with specific expertise in benefit-cost or regulatory analysis, who may have better understandings of the SOC concept than a broader group of economists.

[21] We do not feel that Weitzman's "experts", where in fact experts as they were not selected for discount rate expertise. Our assumption, however, here has some institutional support. The Office of Management and Budget (OMB) has recommended rates ranging from 7 to 10% (US OMB, 2003), while Zerbe and Burgess (2011) estimate the SOC to lie in the range 6 to 8% (we use 5% as our lower bound given the adjustment for the death risk premium as discussed in footnote 19). OMB's recommended rate fell from 10% real during the Reagan administration to

[Insert Figure 1 here]

The Equity Rate starts at 15% and converges to the minimum time preference rate across the *N* citizens, which we assume to be 0%. SOC1 gives OMB's current recommended social discount rate of 7% (US OMB, 2003). SOC2 is equivalent to $-\frac{ln\left(\frac{1}{J}\left(\sum_{j=1}^{J} e^{-SOC_j t}\right)\right)}{t}$ given the assumption of uniformly distributed rates between 5 and 10%. It starts at 7.5% and declines asymptotically to the expert's minimum value of SOC (5%) falling below OMB's 7% recommendation after 35 years. Our recommended social discount rate is represented by the open circles, and is the upper-envelope of the Equity Rate and SOC2 curves. Given our assumed distributions, our recommended social discount rate is equal to the Equity Rate for benefits and costs accruing in the first 28 years of a project and equal to the SOC2 thereafter. Note that if the mean value of citizen's time preference rates is above 7%, then the appropriate social discount rate for benefits and costs in the near-term may be substantially above OMB's recommended rate. We also plot Weitzman's social discount rate, which is substantially lower than our preferred rate.[23] Finally, since at this point we have only discussed giving standing to current citizens, and since discounting is only relevant for benefits and cost accrued during their

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7% real just before the Clinton administration. It remains at 7% (US OMB 2003). In certain conditions, namely intergenerational projects, the rate can be 3% real on the basis of the assumption of a diminishing marginal utility of income over time (Zerbe, 2001). Harrison (2010) shows discount rates ranging from 1 to 15% across 20 countries and international organizations, but these countries and organizations vary in whether they use the SOC or another basis for determining their discount rate.

[22] There does not exist sufficient information in the available literature to estimate the actual distribution of time preference rates. Our assumption is arbitrarily chosen to simply illustrate our recommendation from theory.

[23] If the distribution of citizens' time preference rates matched the distribution of preferred social discount rates in Weitzman's survey of economists (i.e., were gamma distributed with a mean of 3.96% and a standard deviation of 2.94%.), then the Equity Rate and Weitzman's Social Discount Rate would be identical. If so, the Equity Rate would lie completely below SOC2 and thus our preferred social discount rate would simply be SOC2 for all years.

lifetimes (see Equation 1), then the relevant portion of Figure 1 is the years during which current citizens can expect to be alive. Thus, years past 100 in Figure 1 are essentially irrelevant when standing is only given to current citizens.

1.7. *Summary of Results When Standing is Given Only to Current Citizens*

To recap, if standing is given only to current citizens, if the goal is to identify whether the Kaldor-Hicks criterion is met (i.e., whether current beneficiaries of a policy could compensate the current citizens who be otherwise be net losers if the policy is adopted), and if the analyst is only charged with evaluating the policy for the perspective of the present, then the benefit-cost analyst should compute the net present value of the policy using Equation 1, and identify whether the project would be rejected using SOC2 rather than individual time discount factors. Since knowing the joint distribution of discount factors and net benefits is likely impossible, a more practical (although still challenging) approach would be to use the maximum of the SOC2 and the Equity Rate. However, there are two practical challenges that will remain: (1) in some circumstances, the analyst may need to recommend that the policy adopt actual transfers in future years to avoid policy regret, and (2) if discount factors are based on irrational individual hyperbolic discounting, then the policy analyst will need to acknowledge the resulting decision may be based on overly impatient discount factors and lead to potential *ex post* regret. The analysis is substantially more challenging if the analyst is charged with evaluating the citizens' retrospective valuations of the project, which would then require knowledge of the joint distribution of retrospective discount rates and the number of years of life remaining for current citizens. The social discount rate emerging from such an analysis could be increasing or decreasing in time, asymptoting to the minimum individual discount rate in the population.

2. Standing Given to Current and Future Citizens

Giving standing to future citizens adds further complexity. Much of this challenge is already faced in traditional benefit-cost analysis (although often not clearly recognized). It is already standard to consider valuations of future citizens as equal in weight to valuations of current citizens – in fact, traditional benefit-cost analysis makes no distinction between current and future citizens. If one gives standing to future citizens, then we argue that Equation 1 should be rewritten as follows:

$$(12) \quad NPV_{Policy} = \sum_{i=1}^{N_{Current\_Citizens}} \sum_{t=0}^{\infty} F_i(\pi_{it}, Z_{it}, e^{-r_i t} V_{it})$$
$$+ \sum_{j=1}^{N_{Future\_Citizens}} \left[\sum_{t=0}^{B_j} e^{-RB_j} Y_{jt} + \sum_{t=B_j+1}^{\infty} e^{-RB_j} F_j\left(\pi_{jt}, Z_{jt}, e^{-r_j t} V_{jt}\right) \right],$$

where B_j is defined as the year of the future citizen *j*'s birth[24] measured from year 0 (the initiation of the government policy), *R* denotes the social opportunity cost of capital (SOC2) when Y_{jt} or $F_j()$ are positive or the marginal rate of capital productivity when Y_{jt} or $F_j()$ are negative, and Y_{jt}, V_{jt}, and Z_{jt} denote valuations of the project by citizen *j* in the years before, during, and after citizen *j*'s life, respectively.[25] The first term in Equation 12 was previously included in Equation 1, with $N_{Current\_Citizens}$ replacing *N* (but with the same meaning as in Equation 1), and again gives the net present value of the project to citizens who are alive in year 0. The second term in Equation 12 gives the net present value of the policy in year 0 to future citizens (i.e., those who have not yet been born).

The last term in brackets, $\sum_{t=B_j+1}^{\infty} e^{-RB_j} F_j\left(\pi_{jt}, Z_{jt}, e^{-r_j t} V_{jt}\right)$, gives the value of goods received after citizen *j*'s birth discounted back to year 0. Conceptually, this term is similar to Equation 1 for current citizens. The only difference is that we discount these valuations from year B_j back to year 0. When $F_j()$ is positive, the effects of the project in the years after citizen *j*'s birth have positive present value to citizen *j* in year B_j. Yet, future citizen *j* cannot compensate current citizens for their losses (if they exist) in year 0. In effect, future citizen *j* is like a borrower who agrees to pay society back in year B_j to compensate for the "loan" that is made by current citizens in year 0 when they make the decision to invest in the project. The

[24] It is best to think of the year of "birth" as the year when the future citizen is of age to engage in contracts (e.g., 18 years old in most U.S. states). At that point, it becomes practical for the individual to have willingness to accept valuations of the policy.

[25] V_{jt} reflects how much citizen *j*'s WTP in year *t* for a positive policy outcome (or –WTA for a negative policy outcome) in year *t* to occur assuming citizen *j* is alive that year. Z_{jt} is citizen *j*'s valuation in year B_j (in the year of his/her birth) of policy outcomes in year *t* under the assumption that citizen *j* is dead in year *t*. Finally, Y_{jt} is citizen *j*'s valuation in year B_j of policy outcomes in year *t* under the assumption that citizen *j* is not yet born in year *t*. This last concept is discussed in more detail in the text below.

“interest rate” the lenders (current citizens) should charge to future citizen *j* is given by current citizens’ opportunity cost to loan the funds, which is equal to SOC2. In contrast, if $F_j()$ is negative (i.e., the effects of the project in the years after citizen *j*’s birth have negative present value to citizen *j* in year B_j) then we need to compute the amount current citizens’ would need to set aside if they were to be able to compensate future citizens’ for their losses in year B_j. The “interest rate” the lenders (future citizens) should charge to citizens in year 0 is the marginal rate of capital productivity. The logic of this choice is that current citizens could compensate future citizens by investing sufficient funds into the private capital market which would grow at the marginal rate of capital productivity. Thus, for future citizens, the net present value of the project in year *t* is defined by their willingness to accept a fixed payment of $X in year 0 that is invested in private capital and returned to the future citizen in year B_j that compensates citizen *j* for the negative policy outcome occurring.

Finally, note that Equation 12 includes the net present value of policy outcomes to future citizens of policy outcomes that occur before their birth, $\sum_{t=0}^{B_j} e^{-RB_j} Y_{jt}$. The inclusion of this term is the most conceptually challenging element (and likely the most controversial); although we believe it is appropriate and essential. Suppose a policy is being currently considered that will increase polar bear stocks by 10% for each of the next 200 years. If we allow current citizens existence values of polar bears in years after their death (e.g., 150 years hence) (as given by Z_{it} in Equations 1 and 12), then it follows that we should allow for existence values of future citizens in the years after their deaths (e.g., 170 years hence for citizens born twenty years from today) as given by Z_{jt} in Equation 12. By the same logic, if a future citizen can have existence values after their death, they could likewise place value on the size of polar bear stocks in the years before their birth (e.g., 10 years hence) as given by Y_{jt} in Equation 12. Now, we recognize the near impossibility of estimating these future citizen’s valuations of policy outcomes before their births – estimating these valuations using surveys of current citizens to elicit valuations of outcomes before their births seems overly heroic given the conceptual challenge of the hypothetical scenario that would be put forth since the past is already pre-determined. Nonetheless, conceptually, we argue that this existence value of outcomes before birth for *future* citizens is an appropriate value to include as it does reflect actual value to citizens with standing.[26]

The practical and theoretical challenges that exist for Equation 1 likewise exist for Equation 12: knowledge of the joint distribution of time discount rates and project benefits would be difficult to estimate (particularly for future citizens); hyperbolic discounting by current and/or future citizens presents the prospect for policy regret; projects with benefits to some citizens and costs to others in any given year could require actual transfers to occur in year *t* for the benefit-cost analyst to avoid making incorrect judgments in year 0 (i.e., ones that would be regretted in future years); and failure to consider the social opportunity cost of capital could result in policy regret.

3. Conclusion: Practical Advice for the Benefit-Cost Analyst in Light of Elusive Theoretical Constructs

If one believes that the Kaldor-Hicks potential compensation test (or the Pareto Relevant test discussed in footnote 1) should be used to guide benefit-cost analysts' conclusions about the merits of a proposed government policy, then we argue that benefits and costs should be discounted by individual citizen's own time discount factors. Doing so, however, brings on substantial practical challenges of identifying the joint distribution of project benefits/costs and time discount factors, particularly if standing is given to citizens who have not yet been born (or who are too young to elicit such valuations). Additional challenge comes if the analyst wishes to give any weight to current citizen's *retrospective* valuation of the project in future years, which would add the daunting challenge of requiring information about the distribution of retrospective discount factors. Moreover, as we demonstrate in this paper, incorporating such individual discount factors potentially puts the benefit-cost analyst in the undesirable position of recommending a policy be adopted that the analyst knows will be regretted by society in the

[26] One possible simplifying assumption could be to use surveys of current citizens to estimate existence values of outcomes after their deaths, and assume that the distribution of these existence values would match the distribution of future citizen's existence values for outcomes *before and after* their lives. We recognize the tenuous nature of this assumption (and even the incredible challenge of eliciting accurate valuations from current citizens). Thus, we leave it to other scholars to deal with the empirical hurdles our theoretical results generate.

future if either (a) citizens have hyperbolic discounting of future values, (b) if some citizens benefit in a particular year while other citizens face costs and *actual compensation* is not included in the policy design in the future year, or (c) if the effect of individual discounting is to result in a social discount rate (which we term the "Equity Rate") which is below the social opportunity cost of capital. In this case the use of the Equity Rate would be inappropriate. The last of these problems can be solved by never using a social discount rate that is less than the opportunity cost of capital, regardless of individual time preferences.

So, what can a central planner or benefit-cost analyst do in light of these elusive theoretical constructs? The analyst should know that there is not an obviously "correct" way to account for individual heterogeneity in discount factors and there is not currently sufficient evidence on the distribution of such discount factors that would allow the analyst to construct a social discount rate from them. At the present, lacking more consensus and empirical evidence, governmental agencies should not be expected to attempt to consider such citizen heterogeneity. Second, the analyst should stay tuned. We are hoping that our article will prompt subsequent scholars to (a) present well-reasoned arguments for whether benefit cost analyses ought to equally value the future perspectives of current citizens and/or equally value the valuations of future citizens, and (b) to empirically estimate the distribution of prospective and retrospective discount factors. If somewhat of a consensus can be developed regarding these two issues, the analyst may see future scholarship that can sensibly derive empirical estimates of the Equity Rate that can be used in practical analyses. Third, we argue that there are solid theoretical grounds for agencies to use a declining discount rate. Even if it is not feasible or desirable to fully implement the discounting we suggest in Equations 1 and 9, we argue that analysts should take note of the results of Equity Rate that asymptotes towards the time discount rate of the citizen with the lowest time preference rate as t goes to infinity. While Weitzman (2001) recommended a declining social discount rate as a result of uncertainty amongst expert analysts regarding the "correct" discount rate, our Equity Rate (which follows in the spirit of Gollier and Zeckhauser, 2005) declines due to heterogeneity across citizens. The combination of these papers should give more substantiation for the argument that social discount rates should be declining as the time horizon lengthens. Combining these insights, we recommend a social discount rate that is declining in time, and is equal to the declining Equity Rate or the social opportunity cost of capital (which may be declining in time due to uncertainty across experts in the correct value of this opportunity cost), whichever is higher. As we have argued, however, the existence of individual discount rates that are very low is not a sufficient reason to use a social discount rate less than the SOC rate. Further, we do not believe that the long-run social opportunity cost of capital would be 0%, and thus differ with Weitzman's (2001) argument for such a low social discount rate for long-term projects.

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Table 1: Conflict Between Choices Made by a Benefit-Cost Analyst and Voters in a Referendum

	Time Preference Rate	**Present Value of \$1 Received in *t* =**					
Person		**1**	**10**	**50**	**100**	**500**	**1000**
1	1%	***0.99***	***0.90***	***0.61***	***0.3679***	***6.7.E-03***	***4.5.E-05***
2	2%	***0.98***	***0.82***	***0.37***	***0.1353***	4.5.E-05	2.1.E-09
3	3%	***0.97***	***0.74***	***0.22***	0.0498	3.1.E-07	9.4.E-14
4	4%	***0.96***	***0.67***	0.14	0.0183	2.1.E-09	4.2.E-18
5	5%	***0.95***	***0.61***	0.08	0.0067	1.4.E-11	1.9.E-22
6	6%	0.94	0.55	0.05	0.0025	9.4.E-14	8.8.E-27
7	7%	0.93	0.50	0.03	0.0009	6.3.E-16	4.0.E-31
8	8%	0.92	0.45	0.02	0.0003	4.2.E-18	1.8.E-35
9	9%	0.91	0.41	0.01	0.0001	2.9.E-20	8.2.E-40
10	10%	0.90	0.37	0.01	0.0000	1.9.E-22	3.7.E-44
	Sum PV	9.47	6.01	1.53	0.58	6.8.E-03	4.5.E-05
	Average PV	0.947	0.601	0.153	0.058	6.8.E-04	4.5.E-06
	Share of Total Willingness to Pay Coming from Person 1	10%	15%	40%	63%	99.3%	99.995%
	Equity Rate	5.5%	5.1%	3.8%	2.8%	1.5%	1.2%

Note: bold entries reflect individuals who would vote for the policy of paying for the program with a lump sum tax equal to the average present value

Table 2: Social Value of a Project From The Perspective of Various Years
(Assuming individuals prefer money received earlier to money received later)

		Bill's Value of Year 0 Costs	Bill's Value of Year 1 Benefits	Bill's Value of the Project	Martha's Value of Year 0 Costs	Martha's Value of Year 1 Benefits	Martha's Value of the Project	*Social Value of Project*
From the perspective of year:	0	-80	74	-6	-80	99	19.00	*13*
	1	-108	100	-8	-81	100	19.20	*11*
	2	-146	135	-11	-82	101	19.39	*9*
	3	-197	182	-15	-82	102	20	*5*
	4	-266	246	-20	-83	103	20	*0*
	5	-359	332	-27	-84	104	20	*-7*
	6	-484	448	-36	-85	105	20	*-16*
	7	-653	605	-48	-86	106	20	*-28*
	8	-882	817	-65	-87	107	21	*-45*
	9	-1,190	1,102	-88	-88	108	21	*-67*
	10	-1,607	1,488	-119	-88	109	21	*-98*

If Bill and Martha live 2 years, is the project worthwhile?

Average	-111	103	-8	-81	100	19	*11*

If Bill and Martha live 10 years, is the project worthwhile?

Average	-543	503	-40	-84	104	20	*-20*

Note: Assumes Bill and Martha discount money received later using exponential discounting with 30% and 1% rates, respectively.

Table 3: Social Value of a Project From The Perspective of Various Years
(Assuming individuals prefer money received *now* to money received later *or earlier*)

		Bill's Value of Year 0 Costs	Bill's Value of Year 1 Benefits	Bill's Value of the Project	Martha's Value of Year 0 Costs	Martha's Value of Year 1 Benefits	Martha's Value of the Project	*Social Value of Project*
From the perspective of year:	0	-80	74	-6	-80	99	19	*13*
	1	-59	100	41	-79	100	21	*62*
	2	-44	74	30	-78	99	21	*51*
	3	-33	55	22	-78	98	20	*43*
	4	-24	41	17	-77	97	20	*37*
	5	-18	30	12	-76	96	20	*32*
	6	-13	22	9	-75	95	20	*29*
	7	-10	17	7	-75	94	20	*26*
	8	-7	12	5	-74	93	19	*24*
	9	-5	9	4	-73	92	19	*23*
	10	-4	7	3	-72	91	19	*22*

If Bill and Martha live 2 years, is the project worthwhile?

Average	-61	83	22	-79	99	20	*42*

If Bill and Martha live 10 years, is the project worthwhile?

Average	-27	40	13	-76	96	20	*33*

Note: Assumes Bill and Martha discount money received earlier or later using exponential discounting with 30% and 1% rates, respectively.

Table 4: Social Value of a Project From The Perspective of Various Years Using Individual Discounting of Future Perspectives
(Assuming individuals prefer money received *now* to money received later *or earlier*)

		Bill's Present Value of:			**Martha's Present Value of:**			
		Bill's Value of Year 0 Costs	**Bill's Value of Year 1 Benefits**	**Bill's Value of the Project**	**Martha's Value of Year 0 Costs**	**Martha's Value of Year 1 Benefits**	**Martha's Value of the Project**	***Social Value of Project***
From the perspective of year:	0	-80	74	-6	-80	99	19	*13*
	1	-44	74	30	-78	99	21	*51*
	2	-24	41	17	-77	97	20	*37*
	3	-13	22	9	-75	95	20	*29*
	4	-7	12	5	-74	93	19	*24*
	5	-4	7	3	-72	91	19	*22*
	6	-2	4	2	-71	90	19	*20*
	7	-1	2	1	-70	88	18	*19*
	8	-1	1	0	-68	86	18	*18*
	9	0	1	0	-67	84	18	*18*
	10	0	0	0	-65	83	17	*17*

If Bill and Martha live 2 years, is the project worthwhile?

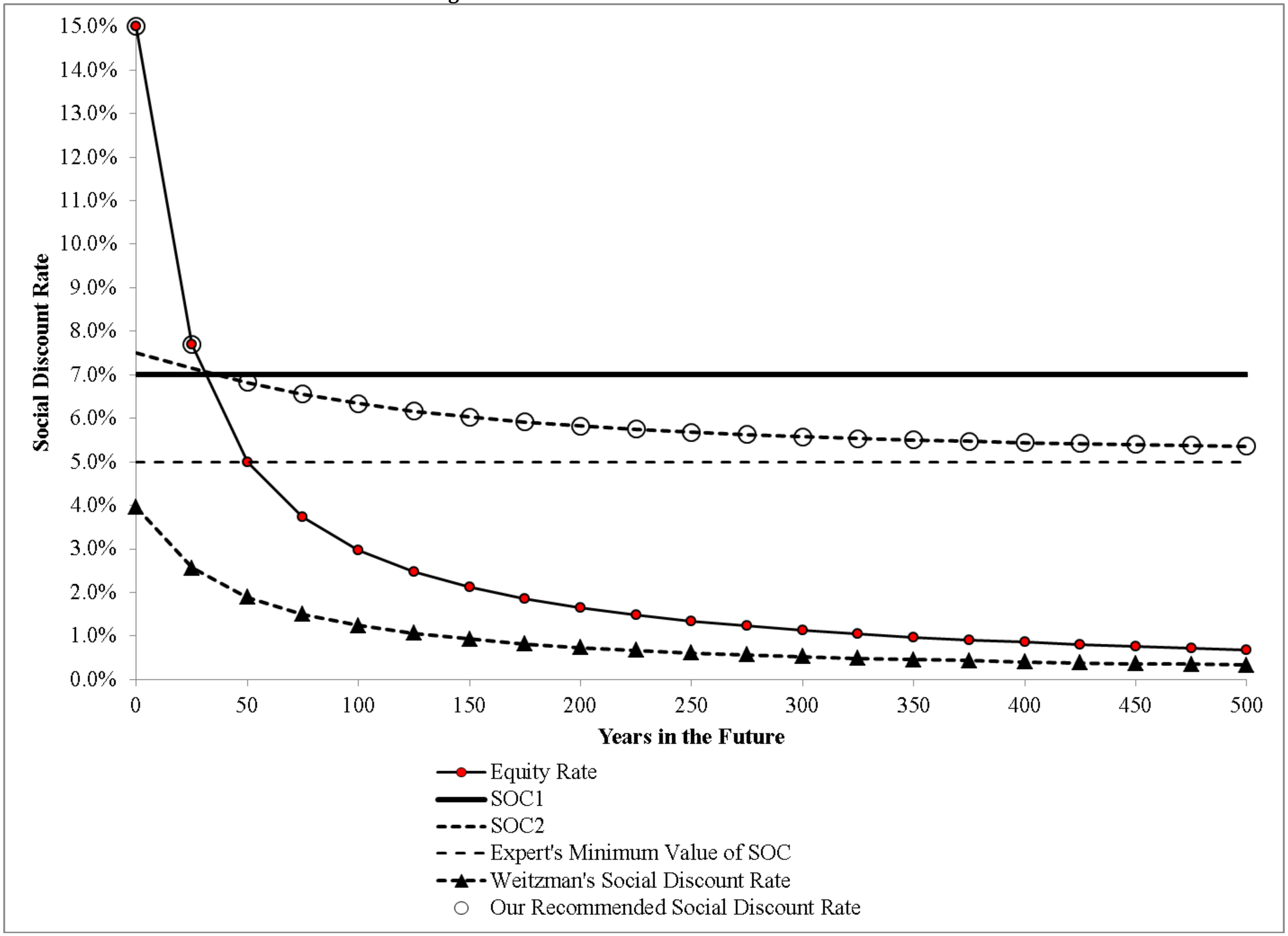
Average	-49	63	14	-78	98	20	*34*

If Bill and Martha live 10 years, is the project worthwhile?

Average	-16	22	6	-73	91	19	*24*

Note: Assumes Bill and Martha discount money received earlier or later using exponential discounting with 30% and 1% rates, respectively. Computes Bill's (Martha's) present value of valuations from the perspective of future years using Bill's (Martha's) discount rate of 30% (1%).

Figure 1: Recommended Social Discount Rate



Appendix: Proof of convergence of the Equity Rate to the time preference rate of the citizen with the lowest preferred time preference rate.

Prove: $\lim_{t\to\infty}(R_t) = \min_{1ii\leq N}\{r_i\}$ where $R_t = \frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t}$

To prove this, we will use the Squeeze Theorem (also known as the pinching or sandwich theorem), which states the following: Assuming that *h(x)* ≤ *f(x)* ≤ *g(x)* and for *x* in an interval around *a* we also have $\lim_{x\to a} h(x) = \lim_{x\to a} g(x) = L$, then $\lim_{x\to a} f(x) = L$. Note that *a* and *L* can be finite or infinite.

Proof:

We have to find an *h(t)* and *g(t)* such that *h(t)* ≤ R_t ≤ *g(t)* and $\lim_{t\to\infty} h(t) = \lim_{t\to\infty} g(t) = L$.

Define $\underline{r} = \min_{1ii\leq N}\{r_i\}$. Then, $r_i \geq \underline{r}\ \forall i$ and $e^{-r_i t} \leq e^{-\underline{r}t}$. Thus, $\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t} \leq \frac{1}{N}\sum_{i=1}^{N} e^{-\underline{r}t}$.

The right-hand side of this inequality can be simplified: $\frac{1}{N}\sum_{i=1}^{N} e^{-\underline{r}t} = \frac{N}{N}e^{-\underline{r}t} = e^{-\underline{r}t}$.

Taking the negative natural log of both sides and dividing by *t* yields: $\frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t} \geq \frac{-\ln(e^{-\underline{r}t})}{t}$.

Simplifying the right-hand side of this inequality, we arrive at Inequality A1:

(A1) $\frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t} \geq \underline{r}$.

Define *h(t)* as equivalent to the right-hand side of Inequality A1. That is, $h(t) = \underline{r}$.

Furthermore, because $\underline{r} \in \{r_i\}\ for\ 1 \leq i \leq N$, we know that $\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t} \geq \frac{1}{N}e^{-\underline{r}t}$.

Again, taking the negative natural log and dividing by *t* yields: $\frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t} \leq \frac{-\ln(\frac{1}{N}e^{-\underline{r}t})}{t}$.

Simplifying the right-hand side of this inequality, we arrive at Inequality A2:

(A2) $\frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t} \leq \underline{r} + \frac{\ln(N)}{t}$.

Define *g(t)* as equivalent to the right-hand side of Inequality A2. That is, $g(t) = \underline{r} + \frac{\ln(N)}{t}$.

From (A1) and (A2), we have

$$h(t) = \underline{r} \leq \frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t} \leq \underline{r} + \frac{\ln(N)}{t} = g(t).$$

Taking the limit yields of *h(t) and g(t)*,

$$\lim_{t\to\infty} \underline{r} = \lim_{t\to\infty} \underline{r} + \frac{\ln(N)}{t} = \underline{r}.$$

Thus, by the Squeeze Theorem,

$$\lim_{t\to\infty} \frac{-\ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{-r_i t}\right)}{t} = \lim_{t\to\infty}(R_t) = \underline{r} = \min_{1ii\leq N}\{r_i\} .$$